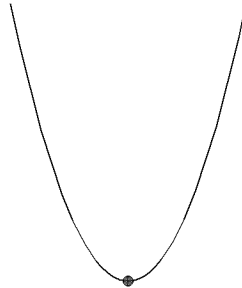


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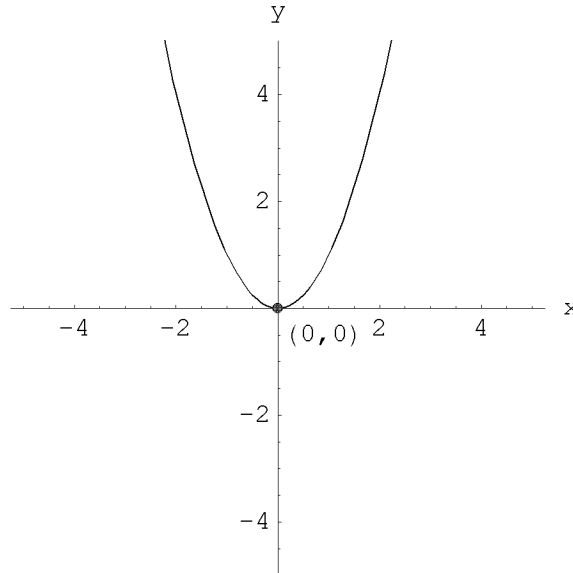
Translation of figures in the plane

■ **It's the coordinate system that moves, not the figure!**

Consider the following parabola. Its vertex is identified by the large dot.

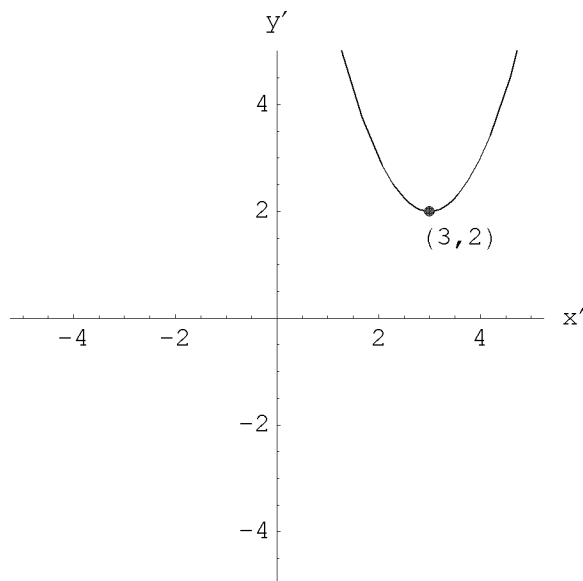


Now overlay a coordinate system. You are free to place the coordinate system where you wish.



In this example, we happen to have placed the origin of the xy system at the vertex of the parabola. In the xy system, the vertex has coordinates $(0, 0)$. The function $y = x^2$ would produce the set of points (x, y) that make up this parabola.

Translate the xy coordinate system 3 units left and 2 units down. Call the shifted coordinate system the $x'y'$ system.



In the $x'y'$ system, the vertex has coordinates $(3, 2)$. The function $y' - 2 = (x' - 3)^2$ would produce the set of points (x', y') that make up this parabola.

In general, if $y = f(x)$ in a certain coordinate system, then the function is written $y' - k = f(x' - h)$, where h and k are positive numbers, when the coordinate system is translated h units left and k units down.

Once this is understood, we typically avoid the correct but cumbersome language " $y - 2 = (x - 3)^2$ is the function $y = x^2$ represented in a coordinate system that has been translated 3 units left and 5 units down from the system in which the vertex lies at $(0, 0)$." Instead, we say " $y - 2 = (x - 3)^2$ is the function $y = x^2$ translated 3 units right and 5 units up."

■ Example 1

The graph of $y - 5 = (x - 1)^3$ is the graph of $y = x^3$ translated 1 unit right and 5 units up.

■ Example 2

The graph of $(x - 7)^2 + (y - 2)^2 = 4$ is the graph of a circle radius 2 centered at $(7, 2)$.

■ Example 3

The graph of $y = (x - 9)^2 + 2$ is the graph of $y = x^2$ shifted 9 units right and 2 units up. [Note: $y = (x - 9)^2 + 2 \iff y - 2 = (x - 9)^2$].

■ Example 4

The graph of $y + 5 = (x + 2)^2$ is the graph of $y = x^2$ shifted 2 units left and 5 units down. [Note: $y + 5 = (x + 2)^2 \iff y - (-2) = (x - (-9))^2$].